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Filamentary structure produced by an electric current in a plasma

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With 1 figure in the text

ABSTRACT

It is shown that in a plasma at low pressure an electric field parallel to a magnetic field produces a current which is confined to a filament. Simple formulae are given for the cross-section of the filament and the current in it.

In many parts of the universe we observe filamentary structures (e.g. prominences, corona filaments, filaments in interstellar clouds). It has been suggested long ago that such filaments may be connected with electric currents, which for some reason are contracted to line currents.[1]

In order to investigate further whether the filamentary structures may have some connection with electric line currents, we shall discuss a very simple stationary case, which seems not to have attracted very much attention so far. We assume that in a cylindrical coordinate system a uniform electric field E is applied parallel to the z-axis. Further we assume that all electric currents $\mathbf{i}(r)$ are symmetric round the z-axis. There is also a magnetic field with a φ component produced by the currents and a z component, H_z, which is in part due to an applied uniform field, H_1, in part produced by the currents. We shall confine ourselves to the case when the mean free path λ of all particles is much larger than the Larmor radius ϱ, which means that we take into consideration only currents parallel to the magnetic field (force free field). The electric parallel conductivity σ is supposed to be constant. We find

$$H_\varphi = \frac{4\pi}{cr}\int_0^r i_z r\, dr \tag{1}$$

and

$$H_z = H_0 - \frac{4\pi}{c}\int_0^r i_\varphi\, dr, \tag{2}$$

where H_0 denotes the field at the z-axis.

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[1] H. Alfvén, Cosmical Electrodynamics, p. 157, 68 (Oxford 1950), and Proc. Roy. Soc. London A, *233*, 296, 1955.

If we denote by θ the angle between a magnetic field line and the z-direction and put $\operatorname{tg}\theta = k$, we have

$$\frac{i_\varphi}{i_z} = \frac{H_\varphi}{H_z} = \operatorname{tg}\theta = k \tag{3}$$

and

$$i = \sqrt{i_z^2 + i_\varphi^2} = \sigma E \cos\theta, \tag{4}$$

which gives

$$\left.\begin{aligned} i_z &= i_0 \cos^2\theta \\ i_\varphi &= i_0 \cos\theta \sin\theta \end{aligned}\right\} \tag{5}$$

and

$$i_0 = \sigma E. \tag{6}$$

We obtain from (1)–(6)

$$\frac{4\pi i_0}{k r}\int_0^r \frac{r\,dr}{1+k^2} + 4\pi i_0 \int_0^r \frac{k\,dr}{1+k^2} = H_0 c. \tag{7}$$

We introduce as a length unit

$$b = \frac{H_0 c}{4\pi i_0} \tag{8}$$

and put

$$x = r/b. \tag{9}$$

Then we have

$$\frac{1}{kx}\int_0^x \frac{x\,dx}{1+k^2} + \int_0^x \frac{k\,dx}{1+k^2} = 1. \tag{10}$$

This integral equation has been solved numerically by Murty. Fig. 1 gives k, i_z, i_φ, H_z, and H_φ as functions of x.

As k increases rapidly with x, the magnetic field at large distances from the axis has almost exclusively a H_φ component, which means that the current goes towards zero. A magnetic field with a finite z-component in infinity is not compatible with a finite value of H_0 (at the axis).

The whole magnetic flux $\phi = \int_0^\infty H_z \cdot 2\pi r\,dr$ can be written

$$\phi = \gamma \pi b^2 H_0, \tag{11}$$

where γ is a constant:

$$\gamma = \frac{1}{\pi H_0}\int_0^\infty H_z \cdot 2\pi x\,dx \approx 3.1. \tag{12}$$

Let us now suppose that we start with a magnetic field H_1 (in the z-direction), which is uniform out to an infinitely conducting cylinder at $r_1 \gg b$. This means that we have a finite magnetic flux $\phi_1 = \pi r_1^2 H_1$ at our disposal. If we apply an electric field in the z-direction, a current starts which finally reaches the stationary state which is depicted in Fig. 1. The stationary state is charac-

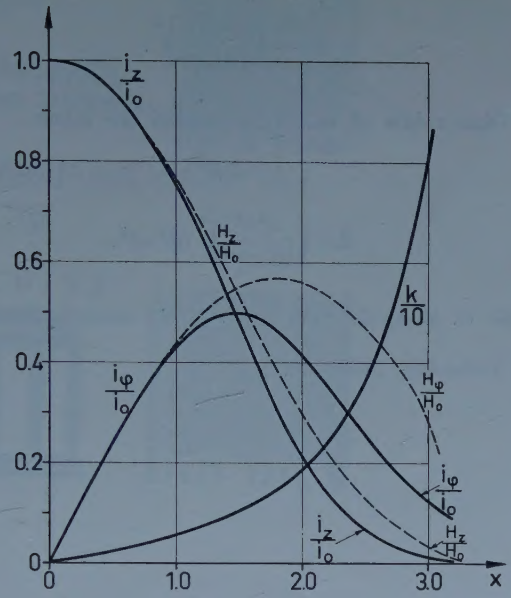

Fig. 1.

terized by the fact that the whole flux, ϕ_1, is concentrated to a cylindrical filament with a radius of the order of b. From (8) and (11) we obtain

$$\phi = \gamma \pi b^2 H_0 = \frac{\gamma c^2}{16\pi} \frac{H_0^3}{i_0^2}. \tag{13}$$

Hence the properties of a filament are completely defined by the available flux ϕ and the maximum current density i_0 (the latter given by the conductivity σ and the electric field E according to (6)). If we express H_0, b, and I as functions of i_0 and ϕ we find:

$$H_0 = \left(\frac{16\pi}{\gamma c^2} i_0^2 \phi\right)^{\frac{1}{3}} \tag{14}$$

and

$$b = \left(\frac{\phi c}{4\pi^2 \gamma i_0}\right)^{\frac{1}{3}}. \tag{15}$$

The total current which the filament carries is

$$I = \int_0^\infty i_z \cdot 2\pi r\, dr = \varkappa \pi b^2 i_0, \tag{16}$$

where $\varkappa$ is a constant:

$$\varkappa = \frac{1}{\pi i_0} \int_0^\infty i_z 2\pi x\, dx \approx 2.6. \tag{17}$$

From (15) we obtain

$$I = \varkappa \left(\frac{c^2 \phi^2 i_0}{16 \pi \gamma^2} \right)^{\frac{1}{3}}. \tag{18}$$

This shows that Ohm's law is invalid. Instead we have

$$I = Z E^{\frac{1}{3}}, \tag{19}$$

with

$$Z = \left(\frac{\varkappa^3 c^2}{16 \pi \gamma^2} \right)^{\frac{1}{3}} (\phi^2 \sigma)^{\frac{1}{3}}. \tag{20}$$

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